TOGGLE PRM: A SIMULTANEOUS MAPPING OF C_{FREE} AND $C_{OBSTACLE}$ FOR USE IN PROBABILISTIC ROADMAP METHODS

An Honors Fellows Thesis

by

JORY LONDON DENNY

Submitted to the Honors Programs Office
Texas A&M University
in partial fulfillment of the requirements for the designation as

HONORS UNDERGRADUATE RESEARCH FELLOW

April 2011

Major: Computer Science

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Approved by:

Research Advisor:	Nancy M. Amato
Associate Director of the Honors Programs Office:	Dave A. Louis

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ABSTRACT

Toggle PRM: A Simultaneous Mapping of
C_{free} and $C_{obstacle}$
for Use in Probabilistic Roadmap Methods. (April 2011)

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Motion planning for robotic applications is difficult. This is a widely studied problem in which the best known deterministic solution is doubly exponential in the dimensionality of the problem. A class of probabilistic planners, called sampling-based planners, have shown much success in this area, but still show weakness for planning in difficult parts of the space, namely narrow passages.

The problem space is made of two subsets - free space and collision space, representing valid and invalid robot positions. A general method for probabilistic planners is the probabilistic roadmap method (PRM) which maps only free space to find a solution. This thesis proposes a new strategy, *Toggle PRM*, for probabilistic roadmap planners, which simultaneously maps both free space and collision space in order to guide the solution more efficiently. All sampled robotic configurations are kept in two separate maps. When the connection attempts between configurations in one roadmap fail, the witness to the failure is retained as a configuration in the opposing roadmap.

By mapping both spaces, sampling density in narrow passages is greatly increased. A theoretical and experimental analysis of Toggle PRM shows the independence

from the volume of a narrow passage and the volume of the obstacles surrounding the passage for sampling, overcoming a previous challenge of probabilistic planning. Additionally, Toggle PRM has increased efficiency as compared to other common sampling techniques in various motion planning problems because of this improved sampling in narrow passages.

DEDICATION

To my family and friends for supporting my academic pursuits.

ACKNOWLEDGEMENTS

Throughout the past year, I have learned much from my mistakes, my experiences, and my friends. This year was a tough year full of struggles which makes this accomplishment all the more worthwhile for me. There are a few individuals who I would like to thank for their help over this past year.

Firstly, I would like to acknowledge the advice given from my mentor. She has been a large influence on my academic career helping me with choosing relevant classes, ensuring progress in research, writing resumes, and applications for graduate school and fellowships. I am very grateful she allowed me to join her research group as a freshman.

Secondly, I would like to thank my friend Jeremy Vu. He is a role model for being a great student and person. I would like to thank him for all the help he gave me with classes and all the pep talks he gave me when times were hard throughout the semester.

Third, Lydia Tapia has been a great advice giver over the past year. And I hope to work with her for many years to come whether it is in graduate school or collaboration.

Lastly, I would like to thank my friends and family for supporting all my life choices and for never telling me that I could not do what I set my mind to.

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CHAPTER I

INTRODUCTION

In robotics, one challenging problem is planning a valid (collision-free) path through an environment. This problem, often referred to as *motion planning* (MP), has applications in domains such as robotics, gaming/virtual reality [1, 2] , computer-aided design (CAD) [3] , virtual prototyping [3, 4] , and bioinformatics [5, 6]. Hence, it is important to find efficient methods that can be used to solve the various applications of motion planning.

Unfortunately, deterministic solutions to motion planning are extremely slow. The best known deterministic solution to motion planning is doubly exponential in the degrees of freedom (DOF) of the robot, or the dimensionality of the problem space. These deterministic algorithms are only able to effectively solve problems with DOF$\leq$ 5. So, the algorithms cannot be applied to problems involving rigid-body robots, such as a free flying cube in a three dimensional space (6 DOF total, 3 movement DOF and 3 rotational DOF).

Sampling-based planners [7] were a major breakthrough in motion planning. These algorithms were able to solve many previously unsolved problems in motion planning, especially for high-dimensional problems. While these methods have been shown to be *probabilistically complete*, narrow passages, or small volume regions of C_{free}, remain difficult for them to map. In particular, it has been shown that the volume of such passages impacts the efficiency of a sampling-based planner [8]. The intuition

This thesis follows the style of the *IEEE Transactions on Robotics and Automation.*

is that the smaller the relative volume of the corridor, the more difficult it is to generate samples in it, and the longer sampling based techniques will take to solve the problem. As discussed in Section II, there have been many variants proposed which aim to address this weakness of sampling-based planners [9, 10, 11, 12]. These samplers try to exploit clearance information or generate samples near obstacles, but none of these are able to sufficiently sample within narrow passages of significantly small volume.

This thesis introduces a new approach to sampling-based planning. In the new method, called Toggle PRM, both collision space, C_{obst}, and free space, C_{free}, are mapped in an integrated, coordinated fashion. In this method, we introduce a new paradigm for probabilistic roadmap planners. Firstly, two roadmaps are constructed, one mapping each space, by retaining all samples, valid or invalid. When the connection attempts between configurations in one roadmap fail, the witness to the failure is retained as a configuration in the opposing roadmap. Collectively, mapping both spaces provides useful insight for narrow passages. The intuition behind this strategy is that each connection attempt (local planner result), whether successful or not, provides important information about the connectivity of both spaces, and moreover, witnesses to unsuccessful connections between configurations in one space provide useful configurations in the other space. For example, a failed connection between two collision configurations on either side of a narrow passage would lead to the discovery of a configuration in the narrow passage. Moreover, the probability of finding such a narrow passage configuration would not depend on the volume of the passage, but only on the fact that a connection was attempted between configurations in the bounding obstacles.

In this thesis, we show that this paradigm shift from mapping the free space only to the coordinated mapping of both free and collision spaces results in significant improvements in the effectiveness and efficiency of sampling-based planners. Particular contributions of this work include:

- Introducing a method, Toggle PRM, that increases the probability of sampling configurations in narrow passages, addressing one of the major challenges of previous sampling-based methods.
- Sketching theoretical arguments that explain this improvement for arbitrary two-dimensional problems.
- Providing experimental results confirming that Toggle PRM sampling in narrow passages is independent of the volume of the narrow passage and of the volume of the obstacles surrounding the narrow passage.

With these insights, this work shows the effectiveness of Toggle PRM, both experimentally and theoretically. By sampling densely in the narrow passage, solutions to problems can be found more efficiently. Additionally, experiments show that mapping both spaces is comparably difficult, meaning that narrowness in C_{obst} is also easily detectable experimentally, supporting the claim of independence of obstacle volume surrounding a narrow passage.

CHAPTER II

PRELIMINARIES AND RELATED WORK

In this chapter, preliminaries and related work are discussed. After describing sampling based planners, an overview of the various known PRM variants are given. Next, work that utilizes collision information to guide sampling is given. These are usually more complex algorithms utilizing machine learning methods. Lastly, methods which model C_{space} to predict collisions are described.

A. Preliminaries

A robot is a movable object whose position and orientation can be described by n parameters, or degrees of freedom (DOFs), each corresponding to an object component (e.g., object positions, object orientations, link angles, link displacements). Hence, a robot's placement, or configuration, can be uniquely described by a point $(x_1, x_2, ..., x_n)$ in an n dimensional space (x_i being the ith DOF). This space, consisting of all possible robot configurations (feasible or not) is called *configuration space* (C_{space}) [13]. The subset of all feasible configurations is the *free* C_{space} (C_{free}), while the union of the unfeasible configurations is the *blocked* C_{space} (C_{obst}). Thus, the motion planning problem becomes that of finding a continuous trajectory for a point in C_{free} connecting the start and the goal configurations. In general, it is intractable to compute explicit C_{obst} boundaries [14], but we can often determine whether a configuration is feasible or not quite efficiently, e.g., by performing a collision detection (CD) test in the *workspace*, the robot's natural space.

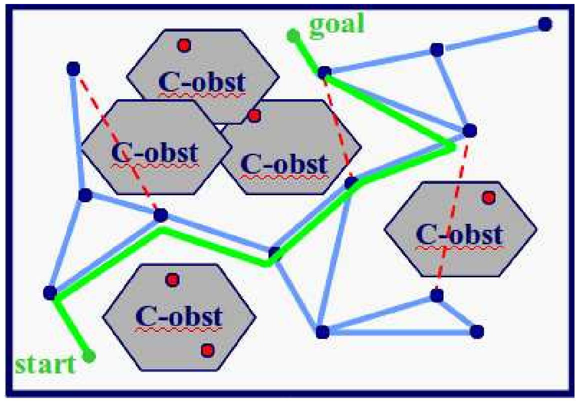


Fig. 1. Example PRM roadmap construction.

Randomized motion planners explore C_{space} and produce a data structure containing feasible configurations and some information about the connectivity of C_{free}. One such planner, the *Probabilistic Roadmap Method* (PRM) [7], builds a roadmap (graph) of the free C_{space}, as in Figure 1. The first phase in this process, *node generation*, is where collision-free configurations are sampled and added as nodes to the roadmap. In the example, valid nodes are shown in blue and invalid nodes are shown in red. In the second phase, *node connection*, neighboring nodes are selected by a *distance metric* as potential candidates for connection. Then, simple *local planners* attempt connections between the selected nodes; successful connections are represented as roadmap edges. In the figure, valid edges are blue lines connecting valid nodes and invalid edges are red dotted lines. Following roadmap construction, queries for valid paths through the environment are examined. In the figure, a start and goal pair can be connected to the roadmap and queried for a path, shown in green.

B. PRM variants

Although the initial PRMs, described in Section A, were successful in solving many problems previously thought unsolvable, they were not successful in problems where the solution path must pass through a narrow passage in C_{space}. In order to address this challenge, many PRM variants have been introduced, and many other heuristics have been proposed [15, 16, 17]. The performance of these methods is dependent on the problem instance. Below is a description of the most relevant PRM variants:

- Obstacle-Based PRM (OBPRM) [9] finds configurations near the surfaces of obstacles in the environment. Firstly, a configuration is sampled uniformly at random. Then, a random direction from the configuration is selected. The configuration is moved along this direction until the configuration changes validity. The valid configuration from this change (before or after depending on the initial validity value) is kept. By this algorithm, configurations are successfully generated near obstacles. However, this algorithm is computationally intensive to find a valid configuration, and the algorithm does not provide any guarantees regarding the distribution of the sampled configurations.

- Gaussian PRM[10] again attempts to find configurations at a Gaussian distance d away from obstacles. First, a configuration is generated uniformly. A second configuration is generated at a Gaussian distance d away along a random directional vector. If and only if one of the configurations is valid and the other configuration is invalid, then the valid configuration is added to the roadmap.

- Bridge Test PRM[12] is similar to Gaussian PRM. It generates a node in collision. Then perturbs this configuration at a small distance λ away on a random directional vector. If the configuration is also in collision, the midpoint between these configurations is taken. The midpoint is kept if and only if it is valid. So in this way, the test attempts to generate configurations in a narrow passage, by bridging the gap, but this test will fail many times before successfully bridging a gap, depending on the narrow passage.

- Medial Axis PRM (MAPRM) [11, 18] samples on the medial axis of the free space. A configuration is generated at random, valid or invalid. Then the configuration is pushed towards the medial axis of the free space based upon its C_{space} clearance. MAPRM significantly improves the quality of the roadmap generated, but it is extremely computationally intensive. Approximations of the clearance can be taken to improve the efficiency. This variant is proven to be more effective than uniform random sampling in sampling in some types of narrow passages.

All the variants are useful in certain cases, each providing their own benefits and weaknesses. Only one of the methods, MAPRM, is proven to be more effective than uniform random sampling for sampling within some types of narrow passages. However, in application, all of the variants improve over uniform random sampling. Most of these methods utilize validity information in order to guide the sampling to aide in finding narrow passages. This work presents another method which aides in generating samples in narrow passages, but is less computationally intensive than some of the variants (e.g., MAPRM).

C. Variants utilizing collision information

There have been many improvements upon PRM by using information gain or learning techniques to guide sampling in specific regions of the environment. Many of these methods utilize information from collisions. However, none of them actually create a map of C_{obst}, but simply use information about nodes in collision. This section describes each of these methods briefly:

- Feature Sensitive Motion Planning [19, 20] applies a supervised learning technique to aid PRMs in solving problems in heterogeneous environments. This work first creates a small roadmap. Regions are recursively identified, broken up, and merged to form homogeneous spaces of the environment. Regions are classified as free, cluttered, narrow passage or blocked based upon an off-line decision tree. Finally, a specific sampler is applied to the region based on this classification. This method utilizes collision information in an entropy based region partitioning scheme[20].

- Region-Sensitive Adaptive Motion Planner (RESAMPL) [21] is a feature sensitive motion planning aid designed to help both multi-query and single-query planning methods. In this work, regions are defined by spheres centered at a node surrounding its nearest neighbors. Both valid and invalid nodes are kept from sampling, and regions are classified by entropy. Regions are labeled as free, surface, narrow, or blocked. All regions are represented in a *region graph* whose nodes are the regions, edges are defined by overlap between regions, and weights are based upon the classification transfer. The information gain and *region graph* bias sampling to improve upon Feature Sensitive Motion Planning [19].

- Exploiting Collision Information in Probabilistic Roadmap Planning [22] proposes a new planning method, which samples uniformly, and classifies each sample as either valid or invalid. If a pair of invalid configurations in C_{obst} are less than a predefined $2r$ distance apart and not from the same obstacle, then the gap between the samples is considered a narrow passage. A region is defined as the centroid between these two points with radius $2r$. Sampling is biased in these centroids.

These methods utilize information from C_{obst}, but none utilize information of the connectivity of the obstacle space, as is done by Toggle PRM.

D. Modeling C_{space}

Other work attempts to model C_{space} [23, 24]. The goal in these methods is reduction in computation time rather than strictly guiding sampling. These are approaches which use a machine learning technique, locally weighted regression, to create an approximate model of C_{space} avoiding unnecessary collision checks. They use their approximate model to bias sampling towards areas which improve knowledge of the model, called active sampling. They create a roadmap, called a predictive roadmap, which uses this model and active sampling for multiple query scenarios. This work requires an accurate and complex model for predicting collisions. Additionally, a utility function guides sampling toward areas which help expand knowledge of the connectivity of C_{space}. The utility function is based on information gain. These methods do well over Uniform random sampling, and significantly reduce the computation time. The bias towards the connectivity of C_{space} is similar to MAPRM.

E. Lazy evaluation

The Toggle PRM algorithm proposed in this work uses a delayed processing of node connection. The delayed evaluation is similar to some heuristics proposed to improve the runtime of PRM methods. Lazy PRM [16] assumes nodes and edges are valid during basic roadmap construction. During query the nodes and edges along the solution path are evaluated for validity. The roadmap is then augmented if no solution is found. Similarly, [25] takes a lazy evaluation scheme to solving manipulation planning problems. In this work, nodes are validated when created for the roadmap, but the edges are assumed possible with a certain probability associated with the feasibility. When querying takes place, the edges are finally evaluated for validity. If no valid path is found the space is augmented with more nodes. Customizable PRM [26] is a lazy-like PRM that saves most edge and node evaluation until query time, limiting the computation of unnecessary nodes and edges. This method utilizes several features that create coarse roadmaps until query time. The coarseness is user defined. At query time, finer resolution of collision detection is utilized, creating an exact, customizable roadmap from start to goal.

CHAPTER III

METHODS

A. Toggle PRM

An overview of *Toggle PRM* is shown in Algorithm 1. In this algorithm, the major difference from traditional PRM methods is the integrated and coordinated mapping of both C_{free} and C_{obst}. Additionally, an alteration to local planning methods is made to return a witness configuration if the connection attempt fails. Both of these allow important information gain to occur that Toggle PRM utilizes to map both spaces efficiently. The algorithm takes in an environment as a motion planning problem, a sampler to generate nodes (e.g., Uniform random sampling), and a connector, which is a combination of a local planner (e.g., straight line) and a nearest neighbor finder (e.g., k closest in a brute force search). We start by initializing two empty roadmaps. While the problem is not solved, then the following happens: samples are generated in both C_{free} and C_{obst}, and repeated connections are attempted in both maps. Failed connections are added to the opposite space's roadmap. If the problem is solved, then the algorithm stops. When repeated connections are attempted, toggling the meaning of validity is necessary for the stopping condition of a local planner to be accurate for the planning space, which is either C_{free} or C_{obst}. The algorithm repeats until the motion planning problem is solved, or shown to be unsolvable.

There are two main benefits of this approach. The first advantage is increased probability of sampling within narrow passages. This is discussed in Section B. Secondly, by using the information gained regarding the connectivity of the spaces,

Algorithm 1 *Toggle PRM.* Local planners and connectors are modified to return configurations from failed connection attempts.

Input: Environment e, Sampler s, Connector c

Initialize Roadmap Graph G_{free} and G_{obst} to $\emptyset$
while $!done$ **do**
  Queue q
  $nodes \leftarrow s.Sample(e)$
  $q.enqueue(nodes)$
  Add free nodes to G_{free}
  Add collision nodes to G_{obst}
  while $!done$ && $!q.isEmpty()$ **do**
    Node $n \leftarrow q.dequeue()$
    if n is valid **then**
      $collisionNodes \leftarrow c.Connect(G_{free})$
      Add $collisionNodes$ to G_{obst} and q
    else if n is invalid **then**
      Toggle Validity
      $validNodes \leftarrow c.Connect(G_{obst})$
      Add $validNodes$ to G_{free} and q
      Toggle Validity
    end if
  end while
end while

i.e., from nodes witnessing failed connections by local planners, the roadmaps of both spaces grow in important regions. Thus, successive iterations of Toggle PRM will expand the roadmap. For example (Figure 2), let sample s be a node within a narrow passage, samples n_1 and n_2 be free samples outside the narrow passage. When a connection between s and n_1 is attempted, a failure configuration x_1 is kept. Similarly when a connection between s and n_2 is attempted, a failure configuration x_2 is kept. x_1 and x_2 are stored in the collision roadmap G_{obst}. When a connection between x_1 and x_2 are attempted, then another free configuration is generated in the narrow passage spatially between s, n_1, and n_2 (red dot in the figure). By repeated iterations, mapping both spaces allows converging on connecting samples within a narrow passage to samples outside the narrow passage.

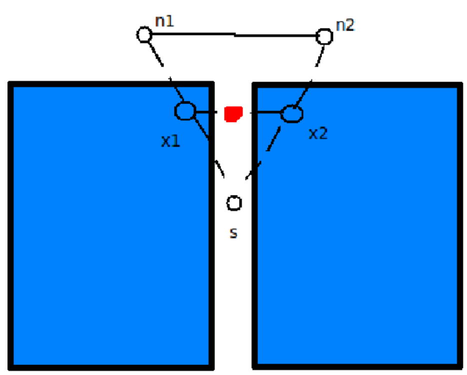


Fig. 2. Toggle PRM example.

We note that once a successful sample is obtained in a narrow passage, the passage itself may be efficiently explored by a tree-based planner such as a rapidly exploring random tree (RRT) [27]. Indeed, many efforts (e.g., [19]) have established that the best planner is in fact an adaptive strategy that selects the best planner for the current situation. This is not the focus of this work however. Here, we focus on understanding this new approach to sampling-based planning.

B. Narrow passages

Narrow passages are notoriously problematic for probabilistic planners. In this section, it is shown that Toggle PRM does not suffer from this low sampling density in narrow passages, and can efficiently find samples in narrow passages regardless of the volume of the narrow passage, or even the volume of the obstacles surrounding the narrow passage.

Proposition 1. *Simultaneous mapping of C_{free} and C_{obst} increases the probability of sampling in a narrow passage over uniform sampling.*

Proof. Let $V(a)$ denote the volume of a narrow passage a. Let $V(c)$ denote the total volume of C_{space}. Let $V(b_1)$ and $V(b_2)$ denote the volume of the obstacles b_1 and b_2 surrounding a. Take two samples. Using uniform random sampling, the probability of yielding at least one sample in the narrow passage is as follows:

$$1 - \left(1 - \frac{V(a)}{V(c)}\right)^2$$

The probability of yielding one sample in the surrounding obstacles is $\frac{V(b_1)}{V(c)}$ and $\frac{V(b_2)}{V(c)}$ respectively. Generating a sample within the narrow passage requires at least one sample in a or two samples must be separately located in b_1 and b_2, so that when a connection is attempted between them a free node will be returned from failure. Thus, the probability to yield a sample in the narrow passage from two samples becomes:

$$P(a) = \left(1 - \left(1 - \frac{V(a)}{V(c)}\right)^2\right) + 2 * \frac{V(b_1)}{V(c)} * \frac{V(b_2)}{V(c)}$$

This is greater than uniform sampling alone. □

From Proposition 1, we can see that the basic idea of simultaneously mapping both

C_{free} and C_{obst} has higher probability than uniform random sampling of successfully sampling in a narrow passage.

Proposition 2. *Toggle PRM is independent of both the volume of a narrow passage and volume of obstacles surrounding a narrow passage.*

Proof. Let a be a narrow passage. From Proposition 1, we know a sample will end up in a narrow passage if either a sample lands in the narrow passage, or two samples separately land in the obstacles surrounding the narrow passage. If we toggle the validity, then probability of yielding a sample in the obstacles is dependent on the free space surrounding the obstacles and the obstacles itself, by Proposition 1. This utilizes all of C_{free} and C_{obst} surrounding the area of a. Thus, toggling the validity and continual sampling makes the behavior of Toggle PRM independent of the volume of the narrow passage and obstacles surrounding it. $\square$

From Proposition 2, it is shown that Toggle PRM utilizes all of C_{space} to generate a sample in a narrow passage.

For comparing Toggle PRM and MAPRM, take Figure 3 as an example of a 2D C_{space}. The narrow passage is labeled A in the figure, which shows the base area of free space of a narrow passage. B and C show the area of the medial axis of the obstacles surrounding the narrow passage from which a configuration will be pushed towards the medial axis of A. D and E show the rest of the obstacle utilized by Toggle PRM. The probability of sampling within this area given two tries is $1-(1-\frac{V(A)}{V(C_{space})})^2$ where $V()$ is a function which calculates the area in the plane. For MAPRM, the probability of generating a sample in the narrow passage given two tries is $1-(1-\frac{V(A+B+C)}{V(C_{space})})^2$. From Proposition 1, the probability of sampling within

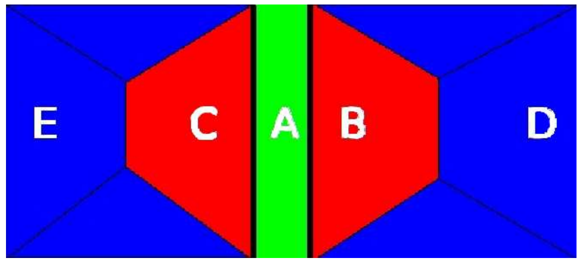


Fig. 3. An environment showing important areas of C_{space} surrounding a narrow passage A.

the narrow passage given two nodes using Toggle PRM is $(1-(1-\frac{V(A)}{V(C_{space})})^2) + \frac{V(B+D)}{V(C_{space})} * \frac{V(C+E)}{V(C_{space})}$, however Proposition 2 shows that the probability of sampling in the narrow passage is approximately 1 as Toggle PRM is independent of these areas over many samples.

CHAPTER IV

EXPERIMENTAL ANALYSIS

A. Experimental setup

In this section, the general testing framework and metrics are described, along with a short description of the experiments shown.

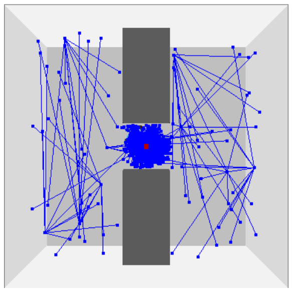
Toggle PRM, MAPRM, and uniform random sampling were implemented using the C++ motion planning library developed by the Parasol Lab at Texas A&M University. RAPID [28] is used for collision detection computations. In these planners, connections are attempted between each node and its k-nearest neighbors according to a distance metric; here we use $k = 5$, C-space Euclidean distance, and a simple straight-line local planner using a binary search method of analysis. The binary analysis is taken so that connection failures are reported quicker on average. Unless otherwise specified, we chose $k = 5$ to keep the computation cost of connections relatively low, MAPRM is implemented as described in [18], Toggle PRM is implemented as described in Algorithm 1, and the queue from the Toggle PRM algorithm is implemented as a priority queue in which valid nodes have priority over invalid nodes for being processed.

For evaluation and comparison of the various planners, three metrics are used. Firstly, the number of free nodes sampled is compared for experiments solving queries. Secondly, the number of CD calls is used as a standard metric of efficiency for experiments solving queries. Thirdly, all experiments compare the percentage of nodes within the narrow passage. A good planner for sampling and solving problems with a narrow passage will ideally minimize the number of free nodes in the roadmap and CD calls while maximizing the percentage of nodes within the narrow passage.

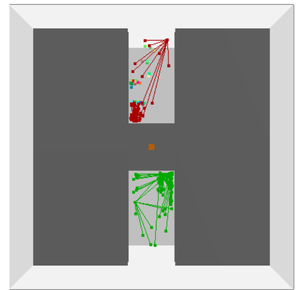
Section B compares the difficulty of mapping both C_{free} and C_{obst}. Section C compares the percentage of nodes obtained in a narrow passage from 1000 total samples by the three methods on three environments with varying narrow passage volumes with constant surrounding obstacle volumes. Section D compares the percentage of nodes obtained in a narrow passage from 1000 total samples by the three methods on three environments with varying obstacle volumes around a constant narrow passage volume. Section E compares the various planners in two 2DOF motion planning problems. These are conditioned to stop when an example query is solved, all three metrics are compared. Lastly, Section 1 shows the extendability of the method to a 6DOF motion planning problem.

B. Comparable sampling difficulty

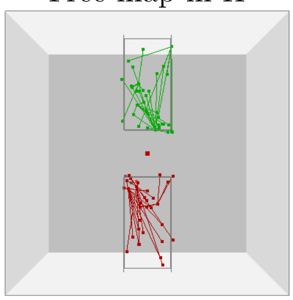
In this experiment, comparable sampling in both spaces is shown. The environments tested and example maps are shown in Figure 4. H environment has the "H" in free space, whereas $\tilde{\mathrm{H}}$ environment has the "H" in block space. 1000 nodes are sampled for the experiments. Toggle PRM is implemented with a regular FIFO (first in, first out) queue. The connection method is to connect to the k closest unconnected nodes not in the same connected component. Failure connections are counted. Here $k = 25$, meaning at most 25 connections are attempted to unconnected nodes. Average number of free and block nodes and percentage of nodes located within the narrow passage of the "H" (valid for H and invalid for $\tilde{\mathrm{H}}$) are averaged over 10 runs and shown in Table I



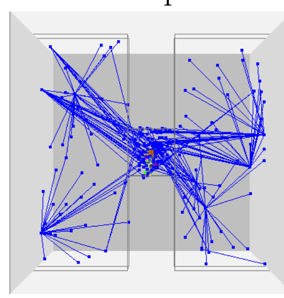
Free map in H



Free map in $\tilde{\mathrm{H}}$



Block map in H



Block map in $\tilde{\mathrm{H}}$

Fig. 4. Example maps of C_{free} and C_{obst} in H and $\tilde{\mathrm{H}}$ environments.

Table I. Results of average free nodes and block nodes along with percentage of nodes within the narrow passage for H and $\tilde{\mathrm{H}}$.

Environment	Free Nodes	Block Nodes	% Nodes in Narrow Passage
H	924	76	78.7
$\tilde{\mathrm{H}}$	108	892	69.8

Firstly, the amount of nodes in the free space of H and the block space of $\tilde{\mathrm{H}}$ are comparable. Similarly, the amount of nodes in the free space of $\tilde{\mathrm{H}}$ and the block space of H are comparable. This is the first step in showing that mapping both spaces is comparable. Next, notice the percentage of free nodes of H and block nodes of H in the narrow passage of the "H" are again comparable, both quite high percentages compared to the proportion of the volume of the passage and obstacle space around this passage, which will be shown more in depth in the following sections. Lastly, the visual placement of nodes in both maps of both spaces are compared to each other, shown in Figure 4. Similar placement of nodes between H and $\tilde{\mathrm{H}}$ are seen. From the maps we see sparsity of nodes in all spaces around the narrow passage of the "H". Also, there is a density of nodes in the narrow passage of the "H". This concludes that both C_{free} and C_{obst} are of comparable difficulty to map in the new paradigm of PRMs introduced here.

C. Insensitivity to narrow passage volume

In this experiment, 1000 samples are taken with each method (valid or invalid) in three simple environments of varying obstacle volumes surrounding a constant narrow passage volume. All cases are 2DOF using a point size robot. The three environments, Passage 1, Passage 2, and Passage 3, are shown in Figure 5. All three of the environments have the same total obstacle volume, 6400 $units^2$, and same total volume of the obstacles which is pushed to the medial axis, 3000 $units^2$. Total C_{space} area is 38000 $units^2$. The various relevant narrow passage volumes are as follows:

- Passage 1. 6400 $units^2$
- Passage 2. 3200 $units^2$.
- Passage 3. 320 $units^2$.

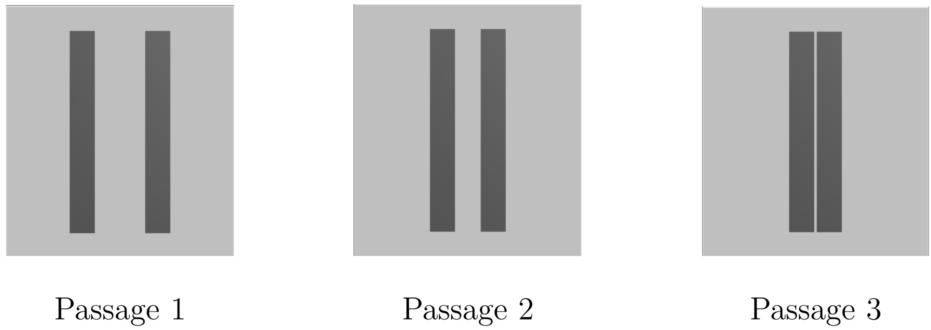


Fig. 5. Simple environments with decreasing narrow passage volume.

From these areas and their respective proportions with all of C_{space}, we expect that over the three environments uniform sampling and MAPRM will yield a decreasing percent of nodes in the narrow passage each time. Toggle PRM is volume independent of the narrow passage, the percentage of nodes in the narrow passage should remain fairly level for the percentage of free nodes. Experiments are averaged over 10 random seeds. Results are shown in Figure 6 and Figure 7.

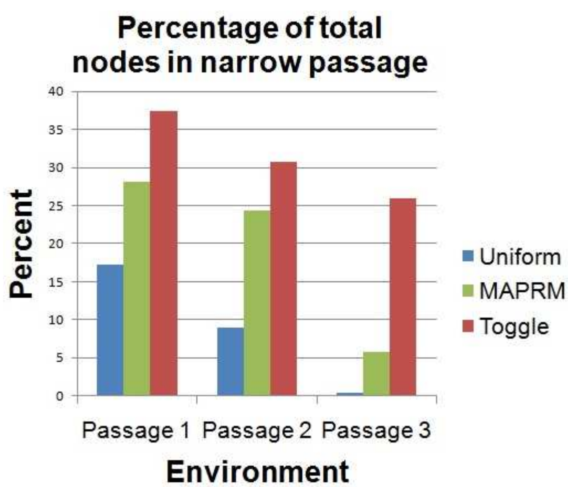


Fig. 6. The percentage of total nodes in the narrow passage in the three environments. Uniform sampling is shown in blue, MAPRM is shown in green, and Toggle PRM is shown in red.

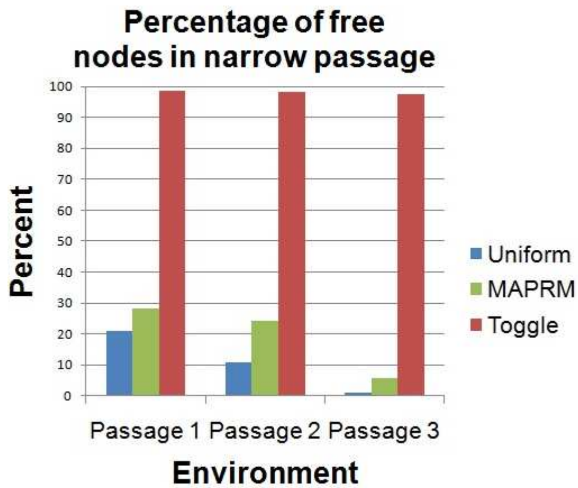


Fig. 7. The percentage of free nodes in the narrow passage in the three environments. Uniform sampling is shown in blue, MAPRM is shown in green, and Toggle PRM is shown in red.

The percentage of total nodes is shown in Figure 6. This is measured as the number of nodes in the narrow passage divided by 1000. Uniform random sampling and MAPRM decreases as the volume of the narrow passage decreases. The percentage of nodes in the narrow passage for Toggle PRM decreases mainly because the total number of free nodes as compared to the number of blocked nodes decreases. The percentage of free nodes in the narrow passage is shown in Figure 7. From this graph, we see that although the number of free nodes is smaller for Toggle PRM, the percentage of free nodes in the narrow passage is significantly greater than both of the other methods.

From these results, Toggle PRM clearly samples densely within a narrow passage even as the volume of that passage decreases. Section D additionally shows the independence of Toggle PRM from the volume of the obstacles surrounding the narrow passage.

D. Insensitivity to surrounding obstacle volumes

In this experiment, 1000 samples are taken with each method (valid or invalid) in three simple environments of varying obstacle volumes surrounding a constant narrow passage volume. All cases are 2DOF using a point size robot. The three environments, Obstacle 1, Obstacle 2, and Obstacle 3, are shown in Figure 8. All three of the environments have the same narrow passage area, 3200 $units^2$. Total C_{space} area is 38000 $units^2$. The various relevant medial axis volumes and obstacle volumes are as follows:

- Obstacle 1. In this environment, total obstacle volume is 3200 $units^2$. The area from the obstacles which is pushed to the medial axis is 1550 $units^2$.
- Obstacle 2. In this environment, total obstacle volume is 6400 $units^2$. The area from the obstacles which is pushed to the medial axis is 3000 $units^2$.
- Obstacle 3. In this environment, total obstacle volume is 19200 $units^2$. The area from the obstacles which is pushed to the medial axis is 7800 $units^2$.

Obstacle 1 Obstacle 2 Obstacle 3

Fig. 8. Simple environments with constant narrow passage volume.

From these areas and their respective proportions with all of C_{space}, we expect that over the three environments uniform sampling will yield the same percent of nodes in the narrow passage each time, medial axis should increase that percentage over the three runs, and since Toggle PRM is approximately C_{obst} volume independent, the percentage of nodes it obtains in the narrow passage should remain fairly level. Experiments are averaged over 10 random seeds. Results are shown in Figure 9 and Figure 10.

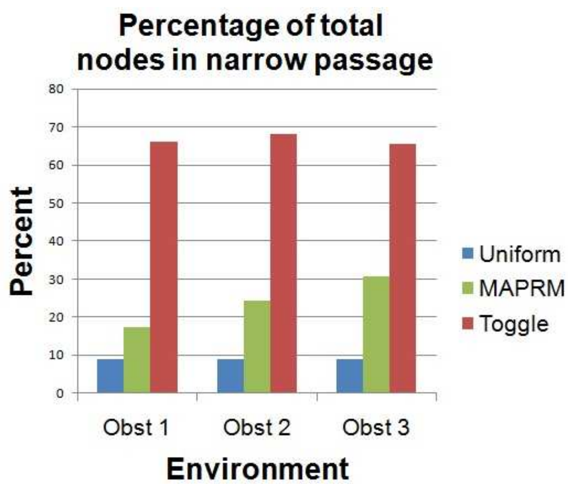


Fig. 9. The percentage of total nodes in the narrow passage in the three environments. Uniform sampling is shown in blue, MAPRM is shown in green, and Toggle PRM is shown in red.

The percentage of total nodes is shown in Figure 9. This is measured as the number of nodes in the narrow passage divided by 1000. Uniform random sampling remains constant in all three cases. MAPRM increases as the proportion of obstacle space that gets pushed to the medial axis increases. Toggle PRM remains fairly level. Additionally we see that the proportion of free nodes gained from Toggle PRM is greater than both uniform sampling and MAPRM by a significant amount in all cases.

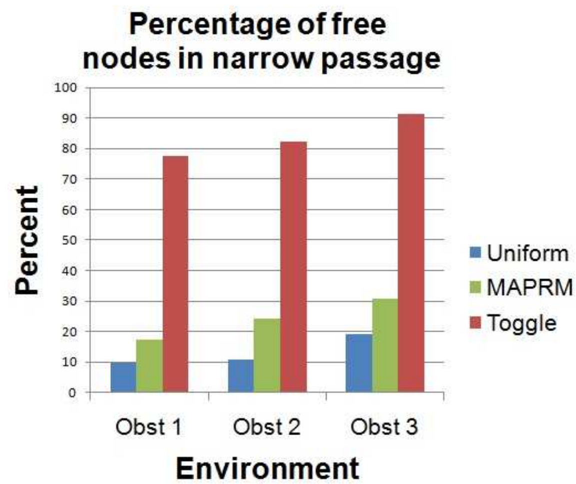


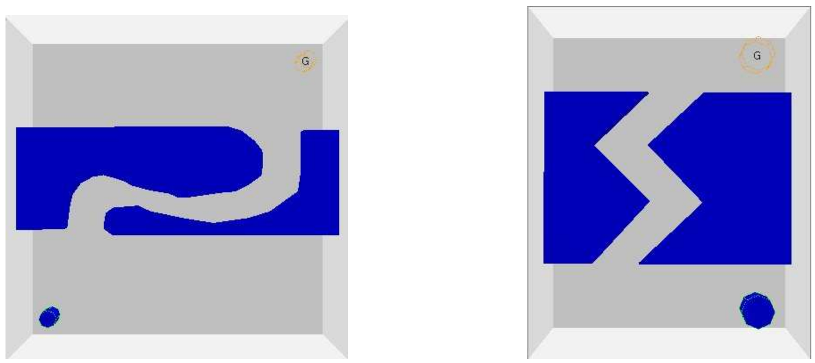
Fig. 10. The percentage of free nodes in the narrow passage in the three environments. Uniform sampling is shown in blue, MAPRM is shown in green, and Toggle PRM is shown in red.

The percentage of free nodes that lie in the narrow passage is shown in Figure 10. Although the number of free nodes produced by Toggle PRM is smaller than MAPRM, the percentage of free nodes in the narrow passage is still higher.

From these results, Toggle PRM clearly samples more densely within the narrow passage. When solving a motion planning problem with a query, Toggle PRM is expected to do well on problems in which a query must traverse a narrow passage, as stated in Section E.

E. Improved problem solving

For this experiment, the three methods are compared against each other in terms of efficient query solving. Here, two problems are analyzed. The environments are shown in Figure 11. Planners are run until the query is solved. Solutions must traverse the narrow passages. Three metrics are compared: number of free nodes sampled, CD calls as a measure of time, and percent of nodes within the narrow passage. Only free nodes are reported for Toggle PRM to compare to the other methods. Data is averaged over 10 runs and shown in Table II.



S2D Zig2D

Fig. 11. Two simple 2D environments. Query solutions must traverse the narrow passage. Robots are shown in the bottom of each figure, they are approximated circles.

Table II. Results for S2D and Zig2D. Toggle PRM is more efficient considering all three metrics.

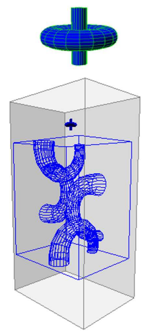
Planner	Free Nodes	CD	% Free in Passage
S2D			
Uniform	1336	14250	8.4
MAPRM	570	1.23e6	19.7
Toggle PRM	**297**	**7582**	**38.2**
Zig2D			
Uniform	1652	9725	5.8
MAPRM	3164	5.44e6	11.5
Toggle PRM	**510**	**8384**	**20.6**

In this experiment, the query was solved in all cases. In both environments, we see that Toggle PRM more efficiently samples the space, with a higher percentage of nodes within the narrow passage. Because of this, it is able to solve a query traversing a narrow passage with fewer nodes and CD calls as compared to uniform sampling and MAPRM. Typically, Toggle PRM doubles the percentage of nodes within the narrow passage compared to MAPRM and quadruples the percentage of nodes within the narrow passage compared to uniform sampling.

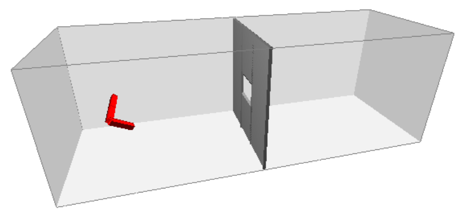
From this experiment, the applicability of Toggle PRM for difficult narrow passages is seen. The benefit of the Toggle method, i.e., mapping both C_{free} and C_{obst}, is gaining more nodes easily within the narrow passage. With a high density of nodes in the narrow passage, the k closest neighbors are more easily connectible. This is because the connection between these nodes has a lower swept volume, and thus a lower chance of colliding with the obstacles. Another benefit of the method is the way upon which it grows a solution utilizing a binary evaluation scheme on the local planner. The middle of the passage is found early, then the roadmap gets expanded from there. This expansion as seen by the results occurs quickly.

1. Higher dimensions

In this section, an experiment using the same general Toggle PRM method is extended to a 6DOF rigid-body robot and an 8DOF articulated linkage robot to show the general extendability of the algorithm. This experiment is evaluated by efficiency in solving a query. The environment is shown in Figure 12. The query must traverse narrow passages in both environments to solve the query. Percentages of free nodes within the narrow passages are approximated by using bounds of the workspace obstacle, as the exact C_{space} bounds of the narrow passages are intractable to compute. Again, only free nodes are reported for Toggle PRM to compare to the other methods. Results are averaged over 10 runs and shown in Table III.



Maze



Hook

Fig. 12. Complex environments where query solutions must traverse the narrow passage. The robot for Maze is a 6DOF rigid-body while the robot for Hook is an 8DOF articulated linkage.

Table III. Results from the Maze problem. Percentage of free nodes in narrow passage is approximated by workspace bounds of the narrow passage.

Planner	Free Nodes	CD	% Free in Passage
Maze			
Uniform	2669	1.00e5	21.1
MAPRM	637	1.98e6	23.7
Toggle PRM	**251**	**5.16e4**	**68.5**
Hook			
Uniform	616	32014	5.0
MAPRM	595	2.24e6	5.2
Toggle PRM	**172**	**10594**	**50.4**

Again, Toggle PRM outperforms the other methods in efficiency. In Maze, Toggle PRM had half the CD calls of Uniform yet approximately tripled both of the other methods in percentage of free nodes within the narrow passage. In Hook, Toggle PRM used only a third of the nodes to solve the problem as compared to uniform sampling and MAPRM. Toggle PRM additionally is able to keep the amount of nodes within the narrow passage ten times that of the other methods. Again, Toggle PRM outperforms the other samplers with fewer CD calls.

CHAPTER V

CONCLUSION

In this paper, we present a new PRM approach which simultaneously maps C_{free} and C_{obst}. The new method, Toggle PRM, successfully improves on the density of sampling within the narrow passage. Additionally, Toggle PRM retains witness nodes from failed local planning attempts. The main advantage to doing this is that when a connection fails in one space, it traverses the other space. Thus, the configuration can be saved and placed in the opposite space's map. Through both theoretical analysis and experimental analysis, the benefit of Toggle PRM is shown to increase the sampling density in a narrow passage. Toggle PRM actually improves not only the percentage of nodes within a narrow passage, but also increases the efficiency of PRM techniques for solving queries. Additionally, it was shown that the performance of Toggle PRM is independent of the volumes of the narrow passage and obstacles surrounding the passage, unlike uniform sampling and MAPRM.

In the future, we plan to explore the applicability of Toggle PRM in higher dimensions, exploring the limitation of this algorithm. Additionally, a few optimizations can be made for both the connections attempted and nodes saved from failed connects. Lastly, an exploration into the applicability of the method in heterogeneous environments, or the development of a divide and conquer approach for these can be taken.

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